HYDRODYNAMICS OF BUBBLER PROCESSES IN THE PRESENCE OF EXTERNAL POTENTIAL FIELDS

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The hydrodynamics of the process in bubble-type reactors in the presence of external potential fields are examined for the case of "slow" bubbling characterized by high weight levels of the two-phase layer and low reduced gas velocities. Equations are obtained for the gas content and specific phase contact surface.

The use of external potential fields makes it possible to intensify chemical engineering processes, modify their motive force and reduce the scale of existing apparatus (in the case of bubbling equipment this is achieved by reducing the theoretical plate number), which considerably reduces thermal losses.

We examine the question of the hydrodynamics of the process in bubble-type reactors in the presence of external potential fields for "slow" bubbling characterized by large weight levels of the two-phase layer and small reduced gas velocities ($F_r \ll 1$) [1]. To determine the gas content in this regime, it is necessary to calculate the size of the bubble in the bubble layer.

It is known [2] that the dynamic interaction of a liquid and a bubbled gas leads to the breakdown and coalescence of gas bubbles, as a result of which energetically more stable bubbles are formed in the bubble layer. The bubble radius can be determined using the equation of relative motion [3]

$$g(\rho_1-\rho_2)V-\xi\frac{\rho_1 U_x^2}{2}\Omega-(\rho_2-\varepsilon\rho_1)V\frac{dU_x}{d\tau}=0. \quad (1)$$

As necessary, by varying the external potential field, we can change both the absolute magnitude and the direction of the quantity g_2. We perform the calculations for a binary mixture, one of whose components has magnetic susceptibility, in the presence of a nonuniform magnetic field. Then,

$$g_2=c_1(x)\frac{\chi_0}{\rho_1}\frac{\partial H^2}{\partial x}. \quad (2)$$

We make the following assumptions. Since the density of the gas bubbled into the liquid is three orders less than the density of the liquid itself, we may assume that $\rho_2 \approx 0$. Assuming that the motion is steady and stabilized, we obtain $dU_x/d\tau = 0$. Multiplying (1) by dx and integrating from 0 to x, we obtain the energy equation for a single bubble in a layer of thickness x. Varying it with respect to r for $r(x) = r_m$ and equating the variation to zero, starting from the minimum energy condition, we obtain

$$r_m=\frac{\xi\rho_1 U_0^2 x}{\rho_1 g_1 x+\dfrac{RT}{v}c_1(x)}. \quad (3)$$

In deriving (3) we disregarded the energy expended on the formation of the bubble surface and made the substitution

$$\int c_1(x)\chi_0\frac{\partial H^2}{\partial x}dx=\frac{RT}{v}c_1(x).$$

It has been established that in mass bubbling the equivalent bubble radius is determined by the physical properties of the medium and over a broad range of gas loads is practically independent of the diameter of the gas-distributing openings or the free cross section of the distributor. The rate at which the bubbles rise is calculated from a formula obtained on the assumption that in our case the flow picture is the same as for a spherical bubble [6]:

$$U_0 \approx \left(\frac{4\sigma^2 g_1}{\alpha\mu\rho_1}\right)^{1/5} = \left[\frac{4\sigma^2}{\alpha\mu\rho_1}\left(g_1 + \frac{c_1(x)}{\rho_1}\chi_0\frac{\partial H^2}{\partial x}\right)\right]^{1/5}, \tag{4}$$

where $\alpha = 12\pi$.

Substituting the value of U_0 from (4) into (3), we can find the mean bubble radius in the bubble layer as a function of the bubble layer parameters.

We now turn to the determination of the gas content of the layer φ and the energy balance. The energy balance for an element of height ∂x is determined as follows:

$$dE_x = dE_1 + dE_2 + dE_3. \tag{5}$$

The quantities dE_1 and dE_2 are found from the equations

$$dE_1 = (1 - \varphi)\rho_1 gxdx, \tag{6}$$

$$dE_2 = \xi\frac{\rho_1 U_x^2}{2}\pi r^2 nxdx. \tag{7}$$

Since the gas flow is continuous, the gas content at the gas-liquid mixture/gas interface is equal to unity [4]. Accordingly, we write

$$\varphi U_0 = U. \tag{8}$$

The volume of the layer element occupied by the gas phase (gas content) is

$$\varphi = \frac{4}{3}\pi r^3 n. \tag{9}$$

Using (8) and (9), we can represent Eq. (7) in the form

$$dE_2 = \frac{3}{8}\frac{\xi\rho_1 U^2}{r\varphi}xdx. \tag{10}$$

The surface tension energy

$$dE_3 = 4\pi r^2\sigma' ndx = \frac{3\sigma'}{r}\varphi dx. \tag{11}$$

Instead of σ' we introduce the mean value (σ_m) of the surface tension

$$\sigma_m = \frac{\sigma + \sigma_{max}}{2}. \tag{12}$$

Equation (12) gives the exact value in the case of a linear dependence of the surface tension on the coordinate. In other cases, it is still possible to use the same formula, since the error is not great owing to the fact that the bubble layers are relatively small.

With these transformations and assumptions, Eq. (5) for the total energy of a two-phase layer of height x_1 takes the form

$$E = \int_0^{x_1}\left[(1 - \varphi)\rho_1 gx + 3/8\frac{\xi\rho_1 U^2}{r\varphi}x + \frac{3\sigma_m}{r}\varphi\right]dx. \tag{13}$$

In this equation the kinetic energy of the two-phase layer has not been taken into account, since it is three orders smaller than the dissipative energy [6].

The equilibrium distribution of the gas content φ corresponds to the case when the energy integral of the layer has a minimum. Using the Lagrange multiplier λ, we vary (13), as a result of which we obtain a functional L in φ [8]. Then, neglecting the variation of x, from (13) we obtain

$$\varphi = \sqrt{\frac{3}{8}\,\frac{\xi\rho_1 U^2 x}{r\left(\frac{3\sigma_m}{r}\;\rho_1 g_1 x - \lambda\right)}}\,. \tag{14}$$

Taking into account the condition that the amount of liquid in the apparatus is fixed, we can determine the height of the two-phase layer x_1, using the expression for φ from (14):

$$H = \int_0^{x_1}\left(1 - \sqrt{\frac{3}{8}\,\frac{\xi\rho_1 U^2 x}{r\left(\frac{3\sigma_m}{r} - \rho_1 g x - \lambda\right)}}\right)dx. \tag{15}$$

The Lagrange multiplier λ is determined from Eq. (14) using the condition $\varphi(x) = 1$. In the general case, in accordance with (3) and (2), which enter into (15), r and g are functions of the x-coordinate and the concentration of the component possessing magnetic susceptibility $c_1(x)$. Consequently, in the final analysis quadrature (15) is determined by the form of $c_1(x)$, which follows from the equation after substitution of the corresponding values of $r(c_1(x), x)$ and $g(c_1(x), x)$:

$$H = \int_0^{x_1}\left[1 - \sqrt{\frac{3}{8}\,\frac{\xi\rho_1 U^2 x}{3\sigma - \dfrac{\xi\rho_1 U^2 x^2\left(\rho_1 g_1 + \dfrac{RT}{v}\,\dfrac{\partial c_1(x)}{\partial x}\right)}{\rho_1 g_1 x + \dfrac{RT}{v}\,c_1(x)} - \dfrac{\lambda\xi\rho_1 U^2 x}{\rho_1 g_1 x + \dfrac{RT}{v}\,c_1(x)}}}\right]dx. \tag{16}$$

We consider the simple case when $c_1(x)$ can be written in the form

$$c_1(x) = c_{\min}(1 + \alpha x), \tag{17}$$

the quantity $c_1(x)|_{x=0} = c_{\min}$ being sufficiently small.

After all the necessary transformations, for the height of the two-phase layer and the gas content we obtain

$$x_1 = \frac{H\sqrt{\xi\rho_1 U^2} + 0.8635}{\sqrt{\xi\rho_1 U^2} + 0.375} \quad \text{and} \quad \varphi_m = \frac{0.8635 - 0.375H}{H\sqrt{\xi\rho_1 U^2} + 0.8635}\,. \tag{18}$$

Even for more general cases of writing $c_1(x)$ it is impossible to isolate x_1 in explicit form. To make an analysis of Eq. (15), we assume that the bubble radius is constant, but less than in the absence of an external field, and also that the acceleration is constant, i. e. , the radius takes the mean value (r_m). Then, after integration, Eq. (15) takes the form

$$\begin{aligned} H = x &+ \frac{U}{2g\sqrt{r_m\rho}}\sqrt{\left(\frac{3\sigma_m}{r_m} - \lambda\right)x - \rho_1 g x^2} - \\ &- \frac{U}{4}\sqrt{\frac{\rho_1}{r_m}}\left(\frac{3\sigma_m}{r_m} - \lambda\right)\frac{1}{(\rho_1 g)^{3/8}}\times \\ &\times\left[\arcsin\frac{x - \left(\frac{3\sigma'}{r} - \lambda\right)\Big/2\rho_1 g}{\left(\frac{3\sigma_m}{r_m} - \lambda\right)\Big/2\rho_1 g} + \frac{\pi}{2}\right]\Bigg|_0^{x_1}. \end{aligned} \tag{19}$$

From (14) with $\varphi(x_1) = 1$ there follows

$$\begin{aligned} \sqrt{\frac{3\sigma_m}{r_m} - \rho_1 g x_1 - \lambda} &= \sqrt{\frac{3}{8}\,\frac{\xi\rho_1 U^2 x_1}{r_m}}\,, \\ \frac{3\sigma_m}{r_m} - \lambda &= \rho_1 g x_1 + \frac{3}{8}\,\frac{\xi\rho_1 U^2 x_1}{r_m}\,. \end{aligned} \tag{20}$$

Thus, the height of the dynamic two-phase layer and the mean gas content are, respectively,

$$x_1 = H\left|\left\{1+\frac{3}{8}\frac{\xi}{r_m}\frac{U^2}{g}-\sqrt{\frac{3\xi}{2}}\frac{U\left(1+\sqrt{\frac{3\xi}{2}\frac{U^2}{4gr}}\right)}{4\sqrt{gr}}\times\right.\right.$$
$$\left.\left.\times\left[\frac{\pi}{2}\arcsin\frac{1-\frac{3\xi}{2}\frac{U^2}{4gr_m}}{1+\frac{3\xi}{2}\frac{U^2}{4gr_m}}\right]\right\}\right|, \tag{21}$$

$$\varphi_m = 1-\frac{H}{x_1}. \tag{22}$$

An analysis of (21) shows that the height of the dynamic two-phase layer increases when external potential fields are applied and, consequently, the mean gas content of the two-phase layer also increases (22).

For gas velocities up to 0.1 m/sec the resistance coefficient is calculated from the formula [7]

$$\xi = 0.82\left[\frac{g\mu^4}{\gamma\sigma^3 q_c^3}\right]^{1/4}\mathrm{Re}. \tag{23}$$

At U > 0.1 m/sec it is not possible to use Eq. (23) and the values of ξ are taken from a graph based on the experimental data on mass bubbling [8].

Using (3) and (22) to calculate the mean bubble radius r_m and the mean gas content of the layer φ_m, we determine the specific phase contact surface

$$a = \frac{S}{V_t}. \tag{24}$$

NOTATION

ρ_1 and ρ_2 are the densities of the liquid and the bubbled gas, respectively; V is the bubble volume; ξ is the coefficient of resistance to the relative motion of the bubble; U_x is the rate of ascent of the bubble in the layer; Ω is the midsection of the bubble; ε is the entrainment factor (for spherical bubbles, $\varepsilon = 0.5$); τ is the time; g_1 is the acceleration of gravity; g_2 is the acceleration due to the external potential field; g is the resultant acceleration; R is the universal gas constant; v is the specific volume of the gas; T is the temperature of the mixture, °K; U_0 is the mean rate of ascent of the bubble in the bubble zone; $c_1(x)$ is the concentration of the component possessing magnetic susceptibility; χ_0 is its magnetic susceptibility for the pure substance; H is the magnetic field strength; μ is the dynamic viscosity of the liquid; E is the total energy of the layer; E_1 is the potential energy of the liquid; E_2 is the dissipative energy of layer; E_3 is the surface tension energy; n is the number of bubbles in the layer; U is the gas velocity at the gas-liquid mixture/gas interface; σ' is the surface tension as a function of the external potential field; σ is the surface tension in the absence of the field; σ_{max} is the surface tension in the region with maximum field strength; γ is the specific weight; V_t is the volume of the two-phase layer, $V_t = V/\varphi_t$; S is the bubble surface; q_c is a conversion factor equal to 9.81.

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